

Formalizing Feature Inheritance

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1 Introduction

Viewing context-free base rules as structure building operations (a rule $S \rightarrow NP VP$ builds an S out of a NP and a VP), the transformational cycle in syntax was a principle that governed the interleaving of transformational operations with context-free structure building operations. In particular, (cyclic) transformational rules were applied only once certain categories (always S , often NP , sometimes PP) of expressions were built. In early minimalism, the transformational rule of movement was interleaved with the structure building operation of merge, however, movement could in principle apply at any time, regardless of the categorial status of its input. Chomsky (2008) has suggested a mechanism of *feature inheritance*, which in effect delays transformations until a particular category is reached. Thus, minimalism with feature inheritance seems to be a return to the original conception of the syntactic cycle.

In this paper we provide a formalization of the mechanism of feature inheritance in the context of minimalist grammars (MGs), itself a formalization of Chomsky's (1995) Minimalist program. The weak generative capacity and worst-case parsing complexity of *feature inheritance* is then compared to that of vanilla MGs.

2 Feature Inheritance

Assume the general clausal architecture of minimalism, where the complementizer head selects the tense head which in turn selects the little- v head which selects the big- V head. A large body of work assumes a shared property between little- v and C ; these two heads are said to define locality domains in the syntax (called *phases*). A basic goal expressed by Chomsky (2008) to reduce the stipulations needed in the theory. As little- v and C share one non-trivial property already, determining whether more of their properties can be identified

would potentially reduce the number of independent stipulations needed to describe the lexicon. Feature Inheritance (FI) is introduced in (Chomsky, 2008) as a way of reconciling a number of related observations with theoretical assumptions, and is made use of by little- v and C , which increases their similarity a great deal.

A main theoretical motivation for FI is to give a larger role to phases. Phases are said to coincide with the portion of the syntactic structure that the interfaces can refer to. In other words, they are the units that semantic and phonological interpretation are defined over. Chomsky suggests that both interfaces refer to the same units of syntactic structure. In addition, he suggests that syntactic operations (like movement and agreement) are not distributed throughout phases, but are rather deferred until the last head in the phase (little- v or C). This desideratum is problematic from the perspective of orthodox analytical assumptions, as the T head is generally considered to trigger movement of and agreement with the surface subject.

One relevant observation is that only finite T heads trigger movement and agreement. A second observation is that the distribution of finite vs nonfinite T is related to the choice of C : for example, the declarative complementizer that selects for finite T , whereas for selects for non-finite T . Chomsky's resolution to the problem is to shift the finite-nonfinite distinction over to C , making T into an underspecified tense head. Then it is C which selects for a generic T head, and it must be C which is responsible for triggering movement and agreement **on T** . FI is the mechanism by which movement triggered by a higher head targets the projection of a lower head, which allows for the idea that movement and agreement is deferred until phase heads are introduced to be realized.

C (and little- v) also permit generic movement to their edges, for example, to break long distance movement into phase-sized chunks. Thus C can

081 trigger movement multiple times, both to its edge,
 082 as well as to the edge of the head immediately
 083 below it. However, the movements that C now
 084 triggers are typically thought to be of two funda-
 085 mentally different kinds: the movement to T is A-
 086 movement, and that targeting C is A-bar-movement.
 087 These kinds of movements have importantly differ-
 088 ent properties (pronouns can be bound after moving
 089 over them with A-movement, but not with A-bar-
 090 movement, for example), and Chomsky (1995) has
 091 proposed that movement steps between the high-
 092 est A-bar position and the lowest base-merge po-
 093 sition of expressions be invisible to various well-
 094 formedness conditions. Making the A and A-bar
 095 movements which C triggers happen simultane-
 096 ously (as opposed to serially) structures the move-
 097 ment dependencies entered into by DPs as trees (or-
 098 dered by derivational order), rather than sequences.
 099 This then eliminates the need to postulate an in-
 100 dependent operation which deletes intermediate
 101 elements in a sequence of movement dependencies
 102 – these are no longer on a single branch of the tree.

Feature Inheritance thus paves the way for
 1. phase heads to be the locus of movement and
 agreement triggers, and 2. a novel approach to the
 distinction between A and A-bar movements.

3 Formal background

We couch our formalization of feature inheritance
 in the formal framework of minimalist grammars
 (Stabler, 1997, 2011), an extensible and well-
 understood grammar formalism capable of transpar-
 ently representing minimalist analyses. Minimalist
 grammars are a lexicalized grammar formalism,
 like categorial grammars, with universal grammat-
 ical rules and complex lexical entries. The cate-
 gories of lexical entries take the form of lists of fea-
 tures, written α or β , called *feature bundles*, where
 a list is a data structure where only the first element
 is directly accessible. Removing ('checking') the
 first element of a nonempty list α results in the re-
 mainder of the list α' (so $\alpha = a.\alpha'$). Features have
 one of two polarities (positive and negative), and
 come in different kinds, represented as different
 names (k , wh , q , d , ...). Two features $+x$ and $-y$
 of opposite polarity *match* iff they are of the same
 kind (i.e. $x = y$).

A syntactic expression is either a pair $\langle w, \alpha \rangle$
 consisting of a string of phonemes w and a feature
 bundle α (written $\mathbf{W}:\alpha$), or a term $\bullet(t_1, t_2)$, where
 t_1 and t_2 are syntactic expressions, and \bullet is either $<$

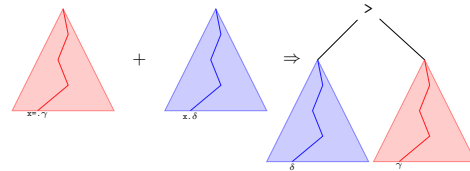


Figure 1: Merge of a specifier

or $>$. The *head* of a syntactic expression t is t itself,
 if a pair, and the head of t_H if $t = \bullet(t_1, t_2)$, where
 $t_H = t_1$ if $\bullet = <$, and $t_H = t_2$ if $\bullet = >$.

Given a syntactic expression t , the result of
 checking the first feature of its head is written t' .
 When t is a term, it represents a tree, and the in-
 ternal nodes 'point' in the direction of the head. A
 trace is a pair of the empty string and the empty
 feature bundle, written t .

There are two syntactic operations, **Merge** and
Move. **Merge** is binary, and **Move** unary. They are
 both restricted in their application by the feature
 bundles present in their arguments. The head of the
 first argument of both operations must be a positive
 feature. **Merge** applies to two expressions t and s
 just in case the heads of both have matching first
 features. **Move** applies to its single argument just
 in case this argument contains a unique leaf whose
 first feature matches the first feature of the head.

The output of **Merge** depends on whether its first
 argument is a leaf or a complex term. If a leaf ℓ ,
 then $\mathbf{Merge}(\ell, s) = \langle \ell', s' \rangle$, and if a proper term
 t , $\mathbf{Merge}(t, s) = \rangle(t', s')$, as is depicted in figure
 1.

Move replaces a subterm of the input with a
 trace, and so we need a notation which simplifies re-
 ferring to subterms. We define *maximal projection*
contexts $C[x]$ to be either a variable x , or a structure
 of one of the two forms: $\rangle(C[x], t)$ or $\langle(t, C[x])$.
 A maximal projection context $C[x]$ is a term where
 x occurs without any arrows pointing to it, and re-
 placing the variable x with a term s is written $C[s]$.
Move applies to t iff $t = C[s]$, where s is a term
 whose head begins with a negative feature which
 matches that of t . $\mathbf{Move}(C[s]) = \rangle(s', C[t'])$, as
 is depicted in figure 2.

Both operations have the effect of removing fea-
 tures from feature bundles one at a time, and fea-
 tures in feature bundles are checked one at a time
 from left to right.

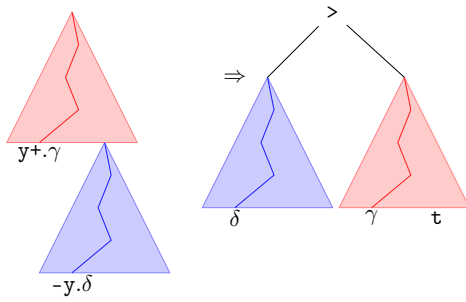


Figure 2: Movement leaves a trace

4 Implementing Feature Inheritance

Feature inheritance diverges from minimalist grammars as they have been defined above in two ways. First, two features can be checked at the same time. Second, movement can target not the top of an expression, but rather some node embedded inside it. To deal with the first difference, we allow feature bundles to contain not just individual features, but also pairs of features. Given a pair of features $\langle +x, +y \rangle$, it is intended that they be checked during the same derivational step. To deal with the second difference, we allow positive features to take a diacritic (written: $+x^\downarrow$) indicating that they should target the sister node to the head. We can augment the **Move** operation so that it can deal with these new feature types. For example, given a term t the first feature of the head of which begins with $+x^\downarrow$, whose complement $C[s]$ contains a unique term s with matching first feature, write $t = D[C[s]]$. Then $\text{Move}(D[C[s]]) = D[\langle s', C[t] \rangle']$.

These operations allow us to write lexical items with the desired behaviour; Chomsky's C head would have feature bundle $+T. \langle +k^\downarrow, +wh \rangle. -C$, indicating that it merges with a TP, then simultaneously triggers **k-movement to TP** and **wh-movement to itself**, and then is itself a CP. However, we are also able to combine pairs and downward diacritics in undesired ways. For example, we could just as easily write the intended $\langle +k, +wh^\downarrow \rangle$ as $\langle +k, +wh^\downarrow \rangle$ or $\langle +k^\downarrow, +wh^\downarrow \rangle$ or as $\langle +k, +wh \rangle$. The intent of Chomsky's proposal seems best captured by the following well-formedness constraint on lexical feature bundles. A lexical feature bundle containing a feature pair $\langle f, g \rangle$ is well-formed if all of the following obtain:

1. There is **exactly one feature pair** in the bundle
2. The feature pair is in the second position in the bundle, immediately following a positive

feature

3. The first feature in the pair has a downward diacritic
4. No other feature in the bundle has a downward diacritic

These constraints are motivated by the following intuitions about Chomsky's proposal.

1. Feature inheritance happens **just once**
2. Feature inheritance happens as soon as the complement is merged
3. Feature inheritance involves transferring features to the head of the complement
4. The only way to transfer features to the head of the complement is via feature inheritance

Taken together, these constraints on feature bundles allow for an alternative implementation of feature inheritance. As feature inheritance targets the first merged argument of the head, and takes place immediately after this argument is merged, it is simple to deal with feature inheritance during this **Merge** step, where the top of the second argument is still accessible. Let ℓ be a lexical item whose feature bundle begins with the following two features: $+x$ and $\langle +y^\downarrow, +z \rangle$. There are two cases to consider, depending on whether one mover matches both features in the pair, or whether they are matched by different movers. For the first case, let $C[s]$ be a term with first feature $-x$, and where the first two features of s are $-y$ and $-z$. Then $\text{Merge}(\ell, C[s]) = \langle (s'', \langle (\ell'', \langle t, C[t'] \rangle)) \rangle$. In the other case, let $C[x, y]$ be a maximal projection context with *two* variables, and let $C[r, s]$ be a term whose first feature is $-x$, and where the first features of r and s are $-y$ and $-z$ respectively. Then $\text{Merge}(\ell, C[r, s]) = \langle (s', \langle (\ell'', \langle (r', C[t, t']) \rangle)) \rangle$.

It only matters that the movement steps be simultaneous if the same mover is targeted in both cases. We thus require that if a single mover can satisfy both features, it must. This is to avoid a situation where two different movers are targeted simultaneously, and then the first one is targeted by a different positive feature, circumventing the intent of simultaneity.

5 Complexity Analysis

Michaelis (2001) proves the equivalence between minimalist grammars and multiple context-free

grammars, providing a scaffolding for future demonstrations that extensions do not increase generative capacity. To establish such an equivalence, we need to present the modified operations in inference rule format, **stated over finite sequences of strings paired with feature bundles**. Our revised implementation of feature inheritance only modifies the **Merge** rule (by adding to it two new cases), and so we present just these in inference rule format (see [Stabler and Keenan \(2003\)](#) for the others). In inference rule notation, to each term corresponds a sequence of string-feature bundle pairs. Each pair beyond the first corresponds to a maximal proper subterm whose head begins with negative features. The first pair corresponds to the term minus these moving pieces.

The inference rule **MrgFI1a** describes the situation where there is a single mover, for whom this is the last movement step, and therefore is pronounced in its highest position.

The inference rule **MrgFI1b** describes the situation where the single mover has features left over, and thus continues moving.

The inference rule **MrgFI2a** describes the situation where there are two movers, for both of which this is the last movement step, and therefore are pronounced in their highest positions. In the result, we see that the phonetic part *o* of the tucking-in mover is sandwiched between the head *m* selecting the complement, and the pronunciation *n* of this complement. The inference rule **MrgFI2b** describes the situation where there are two movers, but there is continuing movement. Pursuant to the discussion above, we only allow targeting two independent movers if the first one does not have multiple features, and thus the only possible continuing mover is the second. As noted by [Stanojević \(2019\)](#), parsers derived from the above inference rule notation can have their worst-case time complexity **read directly off** of the rules themselves. Representing each string as a *span*, a pair of integer variables indicating what portion of the input string that string should cover, the number of distinct variables in the antecedents of a rule polynomially bounds its contribution to worst case complexity. This summation and the associated computational complexity is indicated next to the names of each of the rules above. We see that the rule **MrgFI1b** contributes the most to the worst case time complexity of the new rules. To put this in perspective, the worst case time complexity of minimalist grammars *without* feature inheritance is also $O(n^{2k+3})$

([Fowlie and Koller, 2017](#); [Stanojević, 2019](#)). Thus minimalist grammars with feature inheritance have the same worst case time complexity as vanilla MGs.

6 Conclusion

We have presented a formalization of Chomsky's ((2008)) mechanism of feature inheritance, which has played an important role in minimalist syntactic theory over the intervening nearly two decades. It is formally innocuous: it increases neither the weak generative capacity nor the worst case time complexity of the MG formalism.

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$$\frac{\langle m, +x.(+y^\downarrow, +z).\alpha \rangle \quad \langle n, -x \rangle, \vec{\phi}, \langle o, -y.-z \rangle, \vec{\psi}}{\langle omn, \alpha \rangle, \vec{\phi}, \vec{\psi}} \text{MrgFI1a} \quad \mathcal{O}(n^{2k+2})$$

$$\frac{\langle m, +x.(+y^\downarrow, +z).\alpha \rangle \quad \langle n, -x \rangle, \vec{\phi}, \langle o, -y.-z.\beta \rangle, \vec{\psi}}{\langle mn, \alpha \rangle, \vec{\phi}, \langle o, \beta \rangle, \vec{\psi}} \text{MrgFI1b} \quad \mathcal{O}(n^{2k+3})$$

$$\frac{\langle m, +x.(+y^\downarrow, +z).\alpha \rangle \quad \langle n, -x \rangle, \vec{\phi}, \langle o, -y \rangle, \vec{\psi}, \langle p, -z \rangle, \vec{\chi}}{\langle pmon, \alpha \rangle, \vec{\phi}, \vec{\psi}, \vec{\chi}} \text{MrgFI2a} \quad \mathcal{O}(n^{2k+1})$$

$$\frac{\langle m, +x.(+y^\downarrow, +z).\alpha \rangle \quad \langle n, -x \rangle, \vec{\phi}, \langle o, -y \rangle, \vec{\psi}, \langle p, -z.\beta \rangle, \vec{\chi}}{\langle mon, \alpha \rangle, \vec{\phi}, \vec{\psi}, \langle p, \beta \rangle, \vec{\chi}} \text{MrgFI2b} \quad \mathcal{O}(n^{2k+2})$$